Optimization of maintenance policy using proportional hazard model

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ABSTRACT: The component’s reliability is a function of component age during a system’s operating life. Component aging is strongly affected by maintenance activities performed on the system. Two categories of maintenance activities are considered in this work: Corrective maintenance (CM) and preventive maintenance (PM). We expound a new method to integrate the effect of CM while planning for the preventive maintenance policy. The proportional hazard function was used as a modelling tool for that purpose. Interesting results were obtained.

1 INTRODUCTION

The reliability is an important parameter to assess industrial system performance. Its value depends on the system structure as well as on the component availability and reliability. However, reliability is influenced by maintenance policy. We suggest to study the preventive maintenance (PM), widely applied in large systems such as transport systems, production systems, etc.

These preventive actions can be characterized by their effects on the component age: the component becomes: “as good as new”, the component age is reduced, or the state of the component is lightly affected only to ensure its necessary operating conditions, the component appears to be “as bad as old”. The preventive maintenance corresponds to the maintenance actions that come about when the system is operating. However, the actions which occur after the system breaks down are regrouped under the title of corrective maintenance (CM).

Reaching an “optimal” preventive maintenance policy was the subject of so many studies. Dekker (1996) and Hsu (1999) present surveys over the preventive maintenance models and its evolution. Recently, researchers are more interested in how to optimize those policies. This optimization can be done by adding features and conditions which make this PM policy more realistic: Taking into consideration working conditions, the production schedule of the industry, Perfect and Imperfect actions, etc.

In spite of that corrective maintenance has a direct influence on the component, it was not sufficiently studied.

Under the title of PM optimization, (Tsai et al. 2001) presented periodic preventive maintenance of a system with deteriorated components. Two activities, simple preventive maintenance and preventive replacement are simultaneously considered to arrange the PM schedule of a system. The corrective maintenance effect was only taking into account from the cost point of view. Levitin et al. (2000) and Hsu (1999) have considered the CM as a minimum failure while they were proposing their optimized PM policies.

In this paper the proportional hazard model is used to integrate the effect of the CM on the component’s reliability through its influence on the component’s age.

Kumar et al. (1994) review the existing literature on the proportional hazard model. At first, the characteristics of the method are explained and its importance in reliability analysis is presented. (Martorell et al. 1999) have presented a new age dependent reliability model which includes parameters related to surveillance and maintenance effectiveness and working conditions of the components.
In this paper, the proportional hazard model is used to introduce the CM factor into the component’s reliability and consequently as a decisive parameter in the preventive maintenance planning policy. Comparisons have been established between policies that take into account the effect of the CM and policies which don’t.

This introduction is followed by seven sections which present successively the basic model, the proposed PHM, the system reliability estimation, the cost calculation and the action choice, the case study and finally a conclusion.

2 BASIC MODEL

The “proportional hazard model” is introduced for the first time in 1972 by Dr. Cox. He aimed to consider the effects of different covariates which influence times of component’s failure. Generally the application of the PHM is limited by the case where one treats the replacement of the component by another at the time of repair (as good as new).

In PHM, the failure rate of a component is considered as the product of an initial failure rate depending only on time and a positive function $\phi(z)$. This function reflects the influence of these factors by incorporating covariates which represent them. Thus:

$$h(t,z) = h_0(t)^* \phi(z : \beta)$$  \hspace{1cm} (1)

Let $z$ be an $n \times 1$ vector that contains $n$ covariates which represent the influential factors. $\beta$ is a $n \times 1$ vector of regression coefficients corresponding to those factors.

$\phi(z : \beta)$ should be chosen in a way to satisfy the following conditions:

- $\phi(0)=1$
- $\phi(z) > 0$

Generally, $\phi(z)$ has the following form:

$$\psi(z) = e^{\beta^T z}$$  \hspace{1cm} (2)

$\beta^T$ is the transposed vector of $\beta$. Then, the total failure rate is formulated as following:

$$h(t,z) = h_0(t)\exp(z\beta) = h_0(t)\exp\left(\sum_{j=1}^{n} \beta_j z_j \right)$$  \hspace{1cm} (3)

$z_j, j = 1, 2, \ldots n$ are the considered covariates;

$\beta_j, j = 1, 2, \ldots n$ are the coefficients which define the effect of each covariate.

For more information, interested readers can refer to (Bagdonvicius 2002) and (Kumar et al. 1994).

3 THE PROPOSED PHM

In this paper, Corrective Maintenance (CM) is regarded as an influencing factor. Let $Z_{MC}$ denotes the covariate which represents corrective maintenance and $\beta_{MC}$ the number which defines the size of its influence.

Contrary to what is usually applied to the $\beta$ estimation, we will not calculate a global $\beta$ for all the system, but a local one for each component.

3.1 Effect estimation of corrective maintenance and their number

Let $T_p$ denote the time interval between two preventive maintenance actions. In this interval of time, only two kinds of failures are considered. A type 1 which will be a total failure (catastrophic) and a type 2 which can be repaired by a simple action.

When a failure takes place, it will be a failure of type 1 with a $p(t)$ probability (i.e. probability of failure 1 knowing that the component is failed) and of type 2 with a probability of $q(t) = 1 - p(t)$. In another term, the probability of failure of type 1 is $p_1(t) = p(t) \cdot p_e(t)$ and $q_2(t) = (1 - p(t)) \cdot p_e(t)$ is the type 2 failure probability; $p_e(t)$ is the failure probability.

Since the failure of type 1 is repaired by a replacement and the failure of type 2 is repaired by a simple action, one can simply concludes that replacement will take place with a probability $p(t) \cdot p_{e}(t)$ whereas the simple repair action will take place with a probability of $q_2(t) = (1 - p(t)) \cdot p_e(t)$. Note that those two actions only make the aged component younger by a given improvement factor. This factor is called age reduction coefficient to. Here, the improvement factors, $m_{imp}, m_{par}$ associated to a simple repair act and replacement can take respectively the values of 1 and 0.

Let $m_{MC}(t)$ denote the expected improvement factor of the corrective maintenance at time $t$, it would be calculated as following:

$$p_e(t) \cdot [p(t) \cdot (1 - p(t))] \cdot [m_{imp}, m_{par}]^T = p_e(t) \cdot (p(t) \cdot m_{par} + (1 - p(t)) \cdot m_{imp})$$  \hspace{1cm} (4)

The preceding equation makes it possible to take in consideration that the action taken in a corrective maintenance may vary between the action leading to an “as good as new” state and an “as bad as old” one.

Let $X_j$ denote the $j$th time to failure, $X_i$ can be formulated as following:

$$X_i = \int_{R_{0(i)}}^{T_{r}} \frac{dR(t)}{dt} = \left( \int_{R_{0(i)}}^{T_{r}} R(t) dt \right) + t_{0(i)} R(t_{0(i)}) - T_{r} R(T_{r})$$  \hspace{1cm} (5)
$R(t)$: the component reliability; 
$T_p$: time to preventive maintenance action; 
$t_{0(j)}$: component age after the $j$th corrective maintenance action.

If a CM is performed, the age reduction concept assumes that the effective age $X_j$ is reduced to $t_{0(j)} = m_{CM}(j) \cdot X_j$. During the second interval to CM, the portion of the effective age consumed is $m_{CM}(j) \cdot (X_j - X_{j-1}) = t_{0(j)}$. The improvement by the second CM action does not affect the last $t_{0}$, which is the proportion of the effective age permanently consumed as a result of cumulative damage and environmental effects during the last interval (Tsai et al., 2001).

Thus, we will not only be able to know the number of CM actions taken during a period $T_p$, but their efficiency too.

### 3.2 The proposed model of $\beta$’s calculation

The factor $\beta$ is by definition the factor which reflects the influence of its associated covariate. In our research, covariant is not other than the corrective maintenance. The influence of corrective maintenance can be resumed by the improvement factor $m_{CM}(j)$. Meanwhile, the corrective action time when it was applied has a great importance. That is to say:

$$I_i = \frac{\sum_{j=1}^{N} t_j}{T_i} m_{CM}$$

$t_j$: the time when the $j$th CM action is made. It is equal to $X_j$; 
$T_i$: the time chosen to make the preventive maintenance for the $i$th component; 
$N$: the total number of corrective maintenance; 
$m_{CM}$: the improvement factor for the $j$th CM action.

However, $\sum_{j=1}^{N} \frac{t_j}{T_i} \neq 1$, this is the reason why $I_i$ is multiplied by a corrective factor so that the sum of terms will be equal to 1, the formula will then be:

$$I_i' = \frac{T_i}{\sum_{j=1}^{N} t_j} \cdot m_{CM}$$

$\beta$ must reflect the corrective maintenance impact on reliability. In fact, as much as $I_i'$ is small, the improvement factor is more important, thus reliability must be larger. Knowing that $Z_{MC} = -1$ (improvement factor), consequently $\psi(z) = e^{\beta z}$ must be smaller, thus $\beta = 1 - I_i'$.

### 3.3 The proposed model of $p(t)$'s calculation

Generally, the choice of a maintenance action could depend on the component importance $IF_i(t)$ and the action cost ($C_j$).

We propose to define $p(t)$ as a weight rather than a probability. $p(t)$ could be equal to $f(C_j, IF_i(t))$. $f$ should be chosen in a way that allow to $p(t)$ to approach to 1 as much as the component importance increases and decreases (approach to 0) as much as its importance decreases. $p(t)$ must vary between 0 and 1 to;

$$f(C_j, IF_i(t)) = e^{\frac{C_j Z_{MC}}{IF_i(t)}}$$  

$C_j$: the maintenance action’s cost applied on the $j$th component. 
$IF_i(t)$: the Birnbaum importance factor of the $j$th component at time $t$.

This cost does not only take into account the cost of the action to be made but also the one induced by the unavailability of the system at the time of this action application.

To balance the relative influence of $C_j$ and $IF_i(t)$, we standardize the cost by reporting it with a maximum value $C_{max}$.

### 4 THE SYSTEM RELIABILITY ESTIMATION

The reliability of each component is calculated as following:

$$R(t) = R_0(t) \exp(\beta_{CM} Z_{CM})$$

$$R_0(t) = \exp\left[ -\int_0^t \lambda_0(x)dx \right]$$

$\lambda_0$: the baseline hazard function; 
$Z_{CM}$: the variable which designs the covariate representing the CM; 
$\beta_{CM}$: the factor which defines the CM effect.

In this paper, the studied system is a multi-state one. The estimation of its reliability is based on the Universal Generating Function which presents some advantages in optimization problems.

### 4.1 Multi-state reliability estimation based on a universal moment generating function

The procedure used in this paper for the reliability assessment of a multi-state system is based on the universal moment generating function technique ($\mu$-transform). A detailed description of this method is presented in (Levitin et al., 2000). A brief introduction to the technique is given here.
Consider single elements with total failures. Since each element $j$ has nominal performance $G_j$ and reliability $r_j(t)$, we have:

\[
\Pr(X = G_j) = r_j(t) \\
\Pr(X = 0) = 1 - r_j(t)
\]

Where $X$ designates the state of this single component.

The $u-$function of such element has only two terms and can be defined at time $t$ as:

\[
U_j(z) = r_j(t)z^{G_j} + (1 - r_j(t))z^0
\]

To obtain the $u-$function of a subsystem containing a number of elements, composition operators are introduced. These operators determine the polynomial $U(z)$ for a group of elements connected in parallel and in series, respectively, using simple algebraic operations on the individual $u-$functions of elements. All the composition operators take the form (Levitin et al., 2000):

\[
\Omega(U_1(z), U_2(z)) = \Omega \left[ \sum_{a=0}^{K} \sum_{b=0}^{K} \alpha_a \beta_b z^{a+b} \right] = \sum_{a=0}^{K} \sum_{b=0}^{K} \alpha_a \beta_b z^{w(a,b)}
\]

The function $w(.)$ above expresses the entire performance rate of a subsystem consisting of two elements connected in parallel or in series in terms of the individual performance rates of the elements. The definition of the function $w(.)$ strictly depends on the physical nature of system performance measure and on the nature of the interactions of system elements.

In this paper, we consider that the total capacity of elements connected in parallel is equal to the sum of the capacities of its elements. Therefore:

\[
w(a, b) = a + b
\]

When the components are connected in series, the element with the least capacity becomes the bottleneck of the system. Therefore for a pair of elements connected in series: $w(a, b) = \min(a, b)$. Consequently applying composition operators, we can obtain the $u-$function of the entire system with the general form:

\[
U(z) = \sum_{k=1}^{K} q_k z^{x_k}, \text{ where the variable } X \text{ has } K \text{ possible values. Generally, the definition of multi-state system reliability is defined as } R(t, G_0) = \Pr(G_{sys}(t) \geq G_0) (Levitin et al., 2000), \text{ where } G_{sys}(t) \text{ is the output performance of the system at time } t \text{ and } G_0 \text{ is the required performance i.e. the demand. Therefore, the reliability can be calculated as following: } R(t, G_0) = \Pr(X \geq 0) = \sum_{x_k \geq 0} q_k(t).
\]

5 THE COST CALCULATION AND THE ACTION CHOICE

The cost is equal to the sum of actions costs applied during the preventive maintenance, as well as the production cost lost because of the system unavailability.

The action which maximizes the following expression is chosen as the preventive action to be done:

\[
K = \frac{\left( \int_{T_p}^{T_p} R(t) dt + t_0 R(t_0) - T_p R(T_p) \right) C_{max}}{T_M C_j}
\]

In this expression, the extended life of the component is:

\[
\left( \int_{T_p}^{T_p} R(t) dt + t_0 R(t_0) - T_p R(T_p) \right)
\]

$C_j$ is the cost generated by choosing the $j$th action; $T_M$ is the mission time; $C_{max}$ is the maximum cost that an action could generate.

The $K$ coefficient allows us to balance between the cost and the efficiency of the action (Tsai et al., 2001, 2003).

However we can add some modifications if we want to favorite the price over the life extension or vice versa. The formula would become:

\[
K = \frac{\left( \left( \int_{T_p}^{T_p} R(t) dt \right) + t_0 R(t_0) - T_p R(T_p) \right) C_{max}}{(T_M C_j)^{\beta}}
\]

6 CASE STUDY

To evaluate the impact of the integration of the corrective maintenance effect on the preventive maintenance planning, a comparison is proposed between the costs generated by a maintenance policy taking into account this factor and other ones without this hypothesis. In this latter, the corrective maintenance will influence only the cost since each corrective action is a replacement.

Three structures have been chosen respectively made by 2 and 7 components (see fig. 1, 2 and 3). Each component is characterized by its binary state i.e. the component is either functioning or broken down.

All the occurrence rates (failure and corrective maintenance) of the components in these structures follow an exponential law. Their characteristics are quoted in Table 1. The preventive maintenance action
Figure 1. The first structure.

Figure 2. The second structure.

Figure 3. The third structure.

Table 1. The component characteristics of the three chosen structures.

<table>
<thead>
<tr>
<th>Component number</th>
<th>( \lambda_0 )</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/12.059</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1/12.059</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>1/12.2062</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>1/12.014</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1/191.5197</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>1/63.5146</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>1/66.667</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2. The preventive maintenance action characteristics.

<table>
<thead>
<tr>
<th>Age reduction factor</th>
<th>Action cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>7</td>
</tr>
<tr>
<td>0.7</td>
<td>150</td>
</tr>
<tr>
<td>0.6</td>
<td>200</td>
</tr>
<tr>
<td>0.5</td>
<td>300</td>
</tr>
<tr>
<td>0.4</td>
<td>400</td>
</tr>
<tr>
<td>0.3</td>
<td>500</td>
</tr>
<tr>
<td>0.2</td>
<td>600</td>
</tr>
<tr>
<td>0.1</td>
<td>1000</td>
</tr>
<tr>
<td>Replacement</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table 3. Results comparison (best costs and \( GAPc \)) for two hypotheses (without and with corrective maintenance effect) in the preventive maintenance policy considering three different structures.

<table>
<thead>
<tr>
<th>Components number (structures, cf. Fig. 1, 2 and 3)</th>
<th>( C_{PM} )</th>
<th>( C_{PCM} )</th>
<th>( GAPc )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4000</td>
<td>14</td>
<td>0.996</td>
</tr>
<tr>
<td>7(2)</td>
<td>16341</td>
<td>1341</td>
<td>0.91</td>
</tr>
<tr>
<td>7(3)</td>
<td>16341</td>
<td>1299</td>
<td>0.92</td>
</tr>
</tbody>
</table>

is characterized by an age reduction factor and a cost. The various values of these two factors are quoted in Table 2. Note that during this study we suppose that the time associated to each action is constant.

Let \( GAPc(PM, PCM) = \frac{C_{PM} - C_{PCM}}{C_{PM}} \), where \( C_{PCM} \) is the cost of maintenance generated by a policy taking into account corrective maintenance, and \( C_{PM} \) is the cost of maintenance generated by a policy considering that the corrective maintenance turns the component to, “as good as new”. The results of the comparisons are tabulated in the Table 3. They are obtained with a constraint of reliability equal to 0.8. The mission durations of structures 1, 2 and 3 are respectively 1.5 and 3 years.

We can note the following:

- The \( T_p \) for each component is calculated by using the ant colony system technique elaborated by (Samrout et al., 2005).
- \( GAPc \) shows clearly the importance of integrating the effect of corrective maintenance. However, this \( GAPc \) is not constant for a given system. This remark appears clearly in the \( GAPc \) of structures 2 and 3.
- \( GAPc \) decreases when the number of the components increases. That can be explained by the fact that the preferred choice of action for the components of structure 1 is generally the replacement whereas for the two other structures there are no components of such importance.
- In (Samrout et al., 2005), the action applied during a corrective maintenance is considered to be a minimal whatever the importance of the component is. Even in this case, \( GAPc \) is always significant.

7 CONCLUSION

This paper proposes a new method which allows taking the CM effect in consideration while planning the preventive maintenance policy.

The corrective maintenance effect on the failure rate of the components and consequently on the global system is often neglected. The proposed model offers the possibility to evaluate this effect. The corrective maintenance is no more either minimal repairs or replacements. Thanks to our model they can be any
suitable action to the component regarding its importance and the action’s cost. The established comparison shows the importance of the CM effect on the failure rate and consequently on the adopted PM policy.

Generally, the $\beta$ factor of a proportional hazards model is estimated by maximizing the marginal, partial or maximum likelihood functions which are obtained by considering the contribution to the hazard rate of the individual time to failure (Kumar et al., 1994). In this paper, a linear model is used to estimate $\beta$. We have been based on information gathered (improvement factors) from each component individually so we have a $\beta$ for each element. The UGF is later used to reflect a global impact of all those $\beta$s i.e. the CM. Optimizing this estimation technique seems interesting to seek.

The mathematical model used to calculate $p(t)$ as well as the one adopted to model age reduction are initial propositions. Optimizing those latter is the subject of current studies. Researches carried out by Kijima in virtual age models’ domain would be very helpful (Kijima, 1989).

During this research, we only considered the effect of the CM. The integration of other parameters is interesting to study; we look forward to see the impact of structural and economic dependences effect along with the CM impact on the planning of a PM policy.

REFERENCES


